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K - COMPLEX DETECTION USING THE CONTINUOUS WAVELET TRANSFORM

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ABSTRACT

The wide variety of waveform in EEG signals and the high non-stationary nature of many of them is one of the main difficulties to develop automatic detection system for them. In sleep stage classification a relevant transient wave is the K-complex. This paper comprehend the developing of two algorithms in order to achieve an automatic K-complex detection from EEG raw data. These algorithms are based on a time-frequency analysis and two time-frequency techniques, the Short Time Fourier Transform (STFT) and the Continuous Wavelet Transform (CWT), are tested in order to find out which one is the best for our purpose, being of two wavelet functions to measure the capability of them to detect K-complex and to choose one to be employed in the algorithms. The first algorithm is based on the energy distribution of the CWT detecting the spectral component of the K-complex. The second algorithm is focused on the morphology of the K-complex / sleep spindle waveform after the CWT. Evaluating the algorithms results reveals that a false K-complex detection is as important as real K-complex detection.

Keywords: EEG, K-complex, Short Time Fourier Transform, Continous Wavelet Transform, wave morphology

Introduction

Since the discovery of the Electroencephalogram (EEG) by the German psychiatrist Hans Berger in 1924 extensive studies about electrical activity of the human brain have been carried out. One of these studies correspond to sleep stage classification. In the last twenty years several researches and significance advances have been made in the field of automatic sleep stage classification since it is one of the diagnostic tools needed for assessment of a number of sleep disorders. Automatic sleep analysis is based on the detection of various waveforms in the EEG and other bioelectric signals, and inferring different sleep stages from the detection of these waveforms. However, the strong non-stationarity nature (transient phenomena) of EEG signals has represented one of the main difficulties in the developing of reliable systems for sleep classification.

A non-stationary signal is defined as a short time event whose frequency content vary in time. A traditional analysis technique, for this kind of signals, that provide an image of the frequency contents of a signal as a function of the time is the time-frequency analysis. Several methods or time-frequency distributions can be used, for example the spectrogram (Short Time Fourier Transform) which calculate the power spectrum of the investigated signal seen through a time windows function that slide along the time axis. In this work we will concentrated in another time-frequency distribution, the Continuous Wavelet Transform (CWT). The CWT can be seen as an operator that takes a signal and produces a function depending of two variables: time and scale. In this way the CWT is able to provide information of features corresponding to the signal that are dependent on the scale used. The scale-dependent structure is strongly linked with the frequency content of the signal giving to the CWT a great potential for detecting and identifying signals with exotic spectral features like transients behavior.

Detection of transient signals in Electroencephalograms has been a subject of research for several years. In sleep EEG one of the most relevant transient signals is the K-complex. In literature we have found a sort of methods and algorithms for detection of K-complexes using Neural Networks, feature based approach, independent component analysis, adaptive filters, statistics methods among others. In order to introduce the reader in the K-complex detection field, we will give a brief explanation about some studies which have been carried out in this field.

In this report we will try to probe whether or no using wavelet transform we can improve detection of K-complexes. At the beginning of the last century the Haar transform gave the first step in the wavelet career, but this transform was not very used until early eighties, when geophysicians, theorical physicians and mathematicians developed a solid theory for Wavelet. Since then, Wavelet has been used in several applications, like signal processing, data compress, time-frequency analysis, multirresolution analysis, statistics, vibrations and many others.

In the last fifteen years wavelet has been widely used in EEG analysis as much as epilepsy and Alzheimer diagnosis as sleep stage classification.

The main of this work is to extract information from sleep EEG raw data about the presence of K-complexes. We decided to work in the time-frequency domain instead of either pure time domain or pure frequency domain as previous works in this field. In order to implement a time-frequency analysis the Continuous Wavelet Transform will be employed because it has been probe to be an efficient tool in extraction of transient characteristics from a collection of raw data. Therefore, the problem statement of this work is to build and evaluate a K-complex detection system using the wavelet transform and, posteriorly, evaluate the algorithm performance trying to find out possible important faults that may affect the system.

Relevant Theory

This chapter will try to cover all the necessary theoretical background in order to give the reader a better approach to the sleep stage classification and time-frequency analysis using wavelet transform. It begins with the basic concepts of sleep classification and a brief description of the bioelectrical signal involved, particularly the electroencephalogram (EEG). Then, an explanation of the relevant EEG waveforms is given. As a first step toward a process of EEG transient signal detection, the Joint Time – Frequency Analysis are explained. Finally, a review of the definition and basic proprieties of the Continuous Wavelet Transform, with the corresponding example and reason of why this Transform will be used for time-frequency analysis are given.

1.1 Sleep Analysis

Sleep analysis is a medical tool of vital importance for the diagnosis and treatment of several kinds of sleep disturbance and psychiatric or neurological disorders. Today, a typical study of sleep includes records of the muscle tone (EMG), of the eye movements (EOG) and of the cerebral activity (EEG) although depending on the clinical purpose other physiological parameters like respiration, heart rate, blood pressure, body temperature, hormonal secretions are used. On the basis of such recordings a certain number of sleep stage are distinguished by criteria that have been standardized by general by general agreement (1).

1.2 Electroencephalogram (EEG)

The physiological exploration of sleep involves the study of several signals, such as electroencephalogram (EEG), electro-ocular (EOG), electromyogram (EMG), blood oxygen measurement, temperature and electrocardiogram (ECG). The recording of these signals is called a polysomnogram.

An electroencephalogram represents the activity of bioelectric signals that is determined by the electrical activity of the brain. The oscillations of the brain are called brainwaves. They have certain characteristics including: amplitude ranging between 10-500 µV and frequency between 0.5-40 Hz. For the measurement of brain waves one uses the international standardized system called "International Federation 10-20 system".

The result of a *polysomnogram* is visualized by using a *hypnogram* which distinguishes sleep stages by analyzing successive sequences of 30 seconds of sleep.

These stages are defined by the nature of the signals encountered in the EEG, including the delta, theta, alpha or beta waves, which are identified by their frequency domains.

The main wave types are:

Alpha Research has shown that in a person who is awake, the presence of alpha waves indicates her/his relaxation. Alpha waves range from 8 to 12 Hz, have a nearly sinusoidal shape and an amplitude between 20 and 40 µV.

Beta: When a person responds to external stimulation, alpha waves are replaced by beta waves. They range from 14 to 25 Hz and have an amplitude ranging between 5 and 20 µV.

Theta waves are in the range of 4-8 Hz, with an amplitude of 20-35 µV and they normally occur during sleep but are associated with dream states, creativity and extensive learning possibilities.

Delta: The delta waves range from 0.5 to 4 Hz, have an amplitude greater than 75 µV and occur during deep sleep. According to this classification, the four stages of sleep, represented in *Figure* 1, are defined as follows:

●**Stage 1:** corresponds to moments of decline in the waking state, transition from a relaxed state of wakefulness to sleepiness. Beta waves increase in amplitude (subvigil beta), the percentage of alpha in the posterior (parietal-occipital) lobe falls below 50%, have small amplitude and last about 1-7 minutes, being gradually replaced by theta (theta rhythm).

●**Stage 2:** indicates light sleep. There emerge theta and secondary transient sleeping spindles with 14 Hz frequency, 40 to 50 µV amplitude and no more than one *K complex* with the frequency of 33 Hz, amplitude of 100 µV. In a healthy person they must be present simultaneously in both the left and right hemispheres. Furthermore, the wave magnitude increases significantly in the central brain areas.

●**Stage 3:** indicates slow (stable) sleep, high amplitude slow waves called delta waves (delta rhythm - 75µV), appear on 20-50% of samples, theta waves become irregular. At this stage, transient waves such as sleep spindles and *complex K* still persist

●**Stage 4**: indicates the rapid (paradoxical) sleep state, delta wave increases occur in more than 50% of samples (delta-75µV), theta waves appear regularly, and the secondary waves disappear. Paradoxical sleep or REM (Rapid Eye Movement) gives the impression of a superficial sleep, although the depth of sleep is deeper than in the other stages of sleep (muscle relaxation and rapid eye movements. It is a sleeping phase with dreams, with secretions of growth hormones, having a prevailing role in the restoration function.

Conventionally, the *"slow sleep"* or NREM (Non Rapid Eye Movement) stage groups up stages 1, 2 and 3, while "*fast-paradoxical sleep*" or the REM (Rapid Eye Movement) stage is found in stage 4.

If this classification provides the essential information for identifying certain sleep abnormalities, it leads however to an incomplete classification. Indeed, this classification does not explicitly take into account, for example, the frequency of the occurrences of transient phenomena that are important in pinpointing certain pathologies (figure 2).

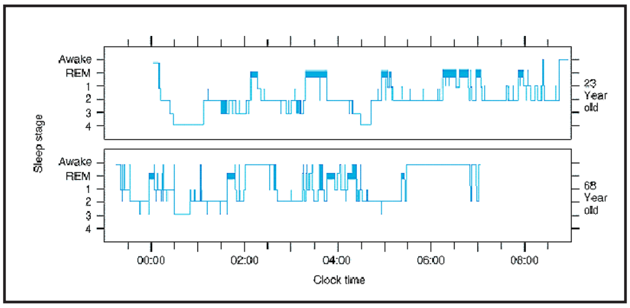

Figure 1: Stage of sleep during the course of the night, for young and elderly subjects.

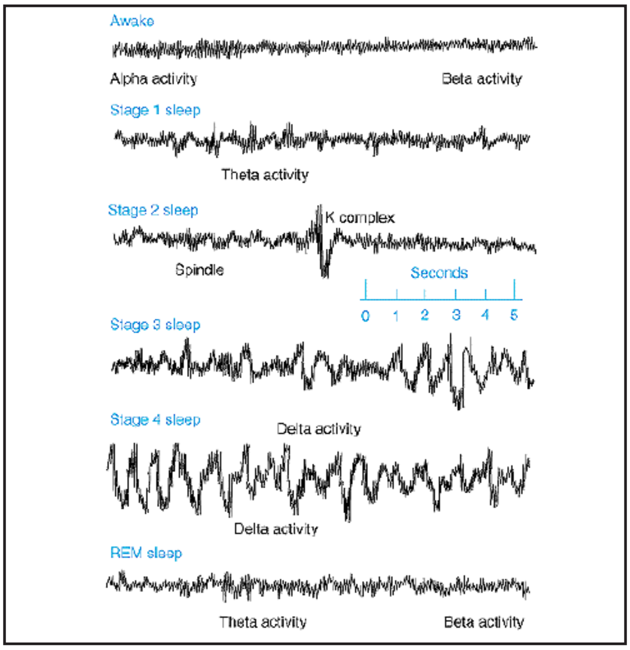

Figure 2: EEG waveforms in various stages of sleep

The K Complex, along with sleep cycles, is one of the main "markers" of the onset of sleep as it appears in stage 2. It is defined by a wave with both polarities, having a minimum amplitude of 100 µVvv, a duration between 0.5 - 1 seconds, preceded and followed by low amplitude activity of no more than 50 µVvv for a duration of at least 2 seconds. Complex K may occur both spontaneously (unevoked) and under the influence of external stimuli. Complex K features a frequency support between 0.5-1.5 Hz for the first peak and 5-10 Hz for the second peak and distinguishes itself by the high amplitude (65µV) of the wave shape of the background electroencephalogram of stage 2 (2,3). *The importance of detecting complex K is due to the significance it hast for prognosis and diagnosis*

1.3 Joint Time-Frequency Analysis

The Cohen -class time-frequency representation

The class of time-frequency representations, in the most general form, starting from the Wigner-Ville distribution relationship, was described by Cohen:

$$C(t,\omega,\Phi)=\frac{1}{2\cdot\pi}\iint\int_{-\infty}^{\infty}\exp[j\cdot(\xi\cdot t-\tau\cdot\omega-\xi\cdot u)]\cdot\Phi(\xi,\tau)\cdot f\left(u+\frac{\tau}{2}\right)\cdot f^{*}\left(u-\frac{\tau}{2}\right)du\,d\tau\,d\xi$$

[1]

where *Φ* is an arbitrary function called kernel function. According to the mode of choosing this function, several particular cases corresponding to certain distributions *(t, ω)* are obtained (4). The most representative time-frequency distributions in the Cohen class are shown in Table 1.

Table 1. Cohen-class frequency-time representations and associated core functions.

$\Phi(\xi,\tau)$	$C_x(t,\nu,\Phi)$	**Distribution**
1	$\int_{-\infty}^{\infty} f\left(t+\frac{\tau}{2}\right)\cdot f^{*}\left(t-\frac{\tau}{2}\right)\cdot e^{-i\cdot 2\pi\cdot\nu\cdot\tau}d\tau$	Wigner-Ville
$\frac{\sin(\pi\xi\tau)}{\pi\xi\tau}$	$\int_{-\infty}^{\infty}\left[\frac{1}{\lvert r\rvert}\int_{t-\frac{\tau}{2}}^{t+\frac{\tau}{2}} f\left(s+\frac{\tau}{2}\right)\cdot f^{*}\left(s-\frac{\tau}{2}\right)ds\right]\cdot e^{-i\cdot 2\pi\cdot\nu\cdot\tau}d\tau$	Born-Jordan
$e^{\frac{-\left(\pi\xi\frac{\tau}{\sigma}\right)^{2}}{2}}$	$\int_{-\infty}^{\infty}\int_{-\infty}^{\infty}\frac{\sigma}{\lvert r\rvert}\cdot e^{-2\sigma^{2}(s-t)^{2}\cdot\tau^{2}}\cdot f\left(s+\frac{\tau}{2}\right)\cdot f^{*}\left(s-\frac{\tau}{2}\right)e^{-i\cdot 2\pi\cdot\nu\cdot\tau}ds\,d\tau$	Choi-Williams
$A_h^{*}(\xi,\tau)$	$\left\lvert\int_{-\infty}^{\infty} f(s)\cdot h^{*}(s-t)\cdot e^{-i\cdot 2\pi\cdot\nu\cdot s}ds\right\rvert^{2}$	Spectrograma

Continuous Wavelet Transform (CWT)

The continuous wavelet transform is used to decompose a signal into wavelets. Wavelets are small oscillations that are highly localized in time. The CWT is an excellent tool for mapping the changing properties of non-stationary signals. The definitions for the CWT are as follows:

$$\int_{-\infty}^{\infty}\psi(t)dt=0 \qquad \psi\in L^{2}(R) \qquad [2]$$

The discrete synthesis operation can be presented as follows:

$$CWT_{\Psi,f}(a,b)=\Psi_{\Psi,f}(a,b)=\frac{1}{\sqrt{a}}\int_{-\infty}^{\infty}f(t)\psi^{*}\left(\frac{t-b}{a}dt\right)$$

[3]

where, $\Psi_{l,k}(a,b)=\left\langle f,\psi_{a,b}(t)\right\rangle$ (5)

Methods and Implementation

2.1 Wavelet Selection

In order to choose the wavelet that will be employed in the K-complex detection algorithm, criteria based on how the wavelet spreads the signal energy in time was developed. Thus, the chosen criteria were based on two main points:

The K-complex frequency has values from 0.5 Hz to 3.5 Hz.

K-complex wave has to have a notorious amplitude difference between the K-complex energy and the energy registered and second before the K-complex and one second after it. This criterion tries to make the distinction between a K-complex and the burst of delta activity.

Based on these criteria, the best wavelet for the detection algorithm will be that which give the biggest difference the energy of the K-complex and the energy calculated and second before and after the K-complex. The first criterion, about the frequency range, was settled using the LabView Based on literature(6,7,8) the most used wavelets for time-frequency analysis have been Mexican Hat and Morlet wavelet. Consequently, these two wavelet were chosen for further analysis. The Mexican hat function is the second derivative of the Gaussian function $e^{-\frac{t^2}{2}}$ and is:

$$\psi = \frac{2}{\sqrt{3}}\pi^{-\frac{1}{4}}(1-t^2)e^{-\frac{t^2}{2}} \qquad [4]$$

The Morlet function is a complex wavelet. The wavelet transform of a real signal with this complex wavelet is plotted in modulus-phase form, however, in this work just the real part will be used. Morlet wavelet is:

$$\psi = e^{-\frac{t^2}{2}}e^{-j5t} \qquad [5]$$

being its real part as:

$$\mathrm{Re}[\psi] = e^{-\frac{t^2}{2}}\cos(5t) \qquad [6]$$

Table 1. Scale range and its corresponding pseudo-frequency range for both Mexican hat and Morket wavelet.

Wavelets	Scale a	Pseudo-frequency [Hz]
Mexican Hat	14-100	3.57 - 0.5
Morlet	43-325	3.53 - 0.5

After determine which wavelet use, the next step was to settle the location in time of the K-complex within its respective 10 seconds epoch signal and its respective time duration T. The K-complex interval T is the value which must be equal or greater that 0.5 seconds and equal or lower than 1.5 seconds (figure 3).

Posteriorly, the CWT was computed and from the absolute values of the obtained coefficients matrix, the highest value in amplitude and its respective frequency value were looked assuming that this frequency is the corresponding spectral component of the K-complex. The wavelet coefficients corresponding only to this spectral component will be called “line of frequency”. Consequently, using the signal extracted from this “line of frequency”, as it is depicted in the right illustration on figure 4.

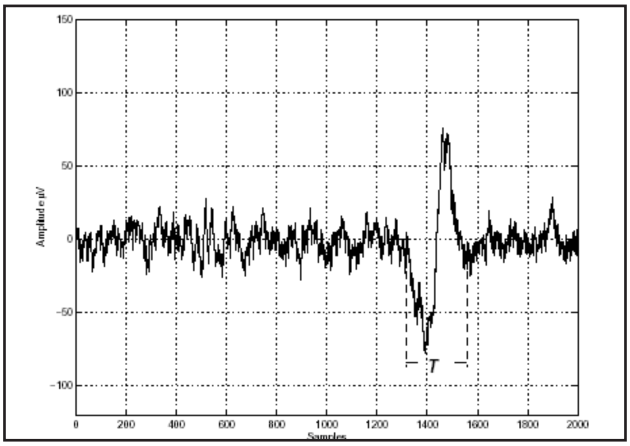

Figure 3. K-complex time period T.

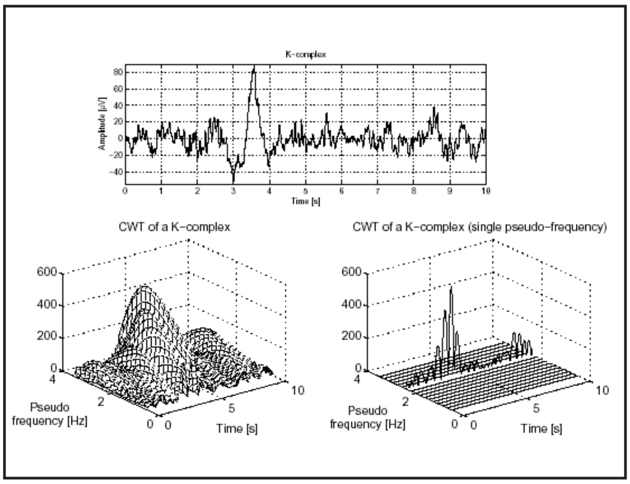

Figure 4. Continuous wavelet transform (absolute value) of the K-complex shown where the maximum amplitude correspond to the pseudo frequency content of the K-complex.

2.2 Algorithm Design

As K-complex are transient phenomena from EEG an algorithm will be developed in order to achieve an automatic detection of these transient signals. The algorithm will be based on time-frequency analysis searching the manner of how quantifies the energy distribution of K-complex in the time-frequency plane. To develop this algorithm the CWT will be employed because this tool has demonstrated a good performance in transient detection and feature extraction in several previous works (9,10). Employing some of the same parameters used in the wavelet selection process, the design of this K-complex detection algorithm will be based on the Energy Distribution of the K-complex in the time-frequency plane using the CWT. The wavelet employed in this algorithm will be the Mexican Hat wavelet function.

As in the wavelet selection procedure, the frequency criterion was based on theory assuming that a K-complex has a frequency range between 0.5 and 3.5 Hz. The pseudo-frequency range obtained was splitted into 17 pseudo-frequency values which were used to calculate the CWT. The scale and pseudo-frequency range are in table 2. The number selected to split the pseudo-frequency range was established basically in order to obtain an acceptable resolution in the time-frequency representation, without compromises the time performance of the algorithm.

Table 2. Scale to frequency transformation using the Mexican Hat wavelet.

Scale	Pseudo-frequency [HZ]
14.00	3.57
19.38	2.58
24.75	2.02
30.12	1.66
35.50	1.41
40.88	1.22
46.25	1.08
51.62	0.97
57.00	0.88
62.38	0.80
67.75	0.74
73.12	0.68
78.50	0.64
83.88	0.60
89.25	0.56
74.62	0.53
100	0.50

The energy distribution criteria were carried out taking a 10 seconds epoch signal with a single clear K-complex and computing the energy value the frequency line belonging to the highest value found in the CWT matrix of that signal. As we defined in the wavelet selection criteria, the pseudo-frequency line corresponding to the highest absolute value in the CWT matrix, will be the K-complex spectral component. This was probed by comparing the Fourier transform of the original signal with the Fourier transform of the frequency line corresponding to the maximum value found in the CWT matrix. As is illustrated in figure 5 we can see that the CWT pseudo-frequency line obtained, the energy per on second was computed having a result of ten energy value per epoch. To calculate the energy per one second E, intervals of 200 samples were taken (because the original signal is sampled at 200 Hz, 1 second contain 200 samples) computing the energy as:

$$E = \sum_{i=1}^{200} \left| s_i \right|^2 , \quad s_i = i \text{ location sample} \qquad [7]$$

Using the K-complex database an attempt to find a common behavior of the energy in the presence of a K-complex was tried.

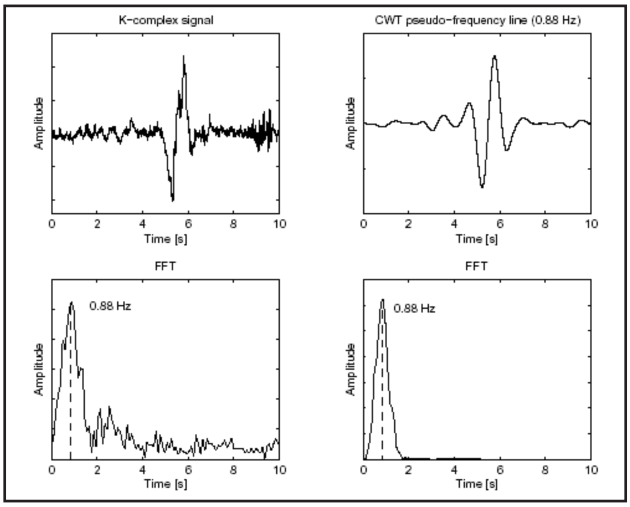

Figure 5. Left – top: K-complex in a ten seconds epoch from EEG; Right – top: CWT for scale 57.00 that correspond to the pseudo-frequency of 0.88 Hz, it can be seen how the wavelet try to assimilate the shape of the K-complex. From this signal the energy value was computed; Left – bottom: Fourier transform of the K-complex, the highest, amplitude correspond to 0.88 Hz; Right – bottom Fourier transform of the CWT pseudo-frequency line.

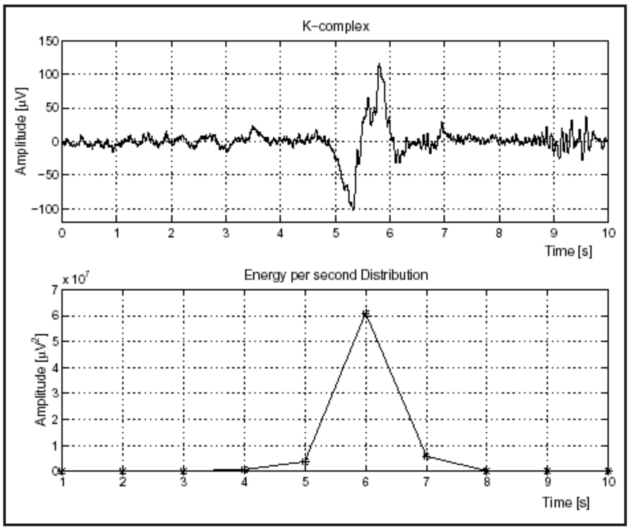

Figure 6. Energy distribution

Results and Discussion

After finish the experimental test, the algorithm performance was tested using the entire eight hours EEG signal (channel 4 signal corresponding to the record position Fp2-M1). Before start the test, a new visual selection of K-complex was made. In this classification we scored 235 K-complex along the entire night. Before run the algorithm through the entire night EEG signal, the obtained results were not as satisfactory as we expect. A total number of 955 event were detected as k-complex. From the 235 previously identified K-complexes, a number of 179 K-complex were detected and 56 were not detected. Therefore, based on this results a total number of 776 false K-complexes were classified as K-complexes by the algorithm. The summarized results can be seen in Table 3 and in Figure 7 and 8.

Table 3. Results of the algorithm performan

Total of detected events	955
Real K-complexes detected	179
Real K-complexes not detected	56
False K-complexes scored	776

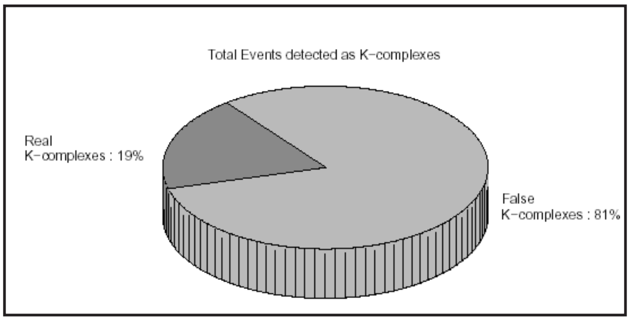

Figure 7. Pie chart plot that shows the percentage distribution of table 3 (discrimination between K-complexes and other transient signal).

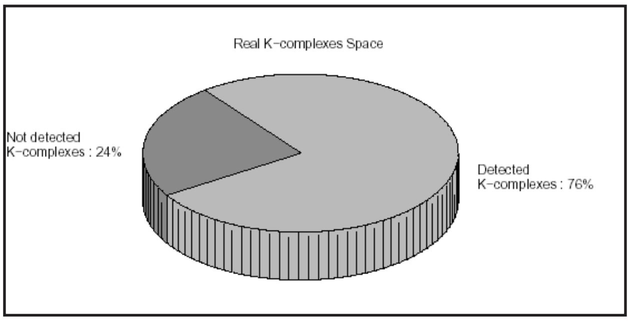

Figure 8. Pie chart plot that describe the percentage distribution achieved in the detection of real K-complexes only.

The algorithm performance has a good capacity to exclude false K-complexes, but the main idea of obtaining a good K-complex detection algorithm, and at the same time, trying to minimize the number of criteria used for the detection was to much restrictive in the criteria number.

Conclusion

In this report we tried to cover the necessary theoretical and practical topics in order to develop different algorithms based on the Continuous Wavelet Transform for K-complex detection on EEG signals. A description of the sleep stage classification, Fourier Transform, Short Time Fourier Transform and Continuous Wavelet Transform was given. The STFT and the CWT are two different tools with the same aim: time-frequency analysis. Are their performance are also different. Therefore, when time-frequency analysis is required, we should be very careful about the features of the signal to analyze, since for some signals the STFT could be more appropriate than the CWT and also in the other direction. For example in signals with no transient content and a limited band width, the STFT has a good performance and the computation time is not large, but when there are transient signals involved, the CWT becomes necessary, and the computation time increase. Two wavelets function were tested with the purpose to obtain a quantitative description about how these two different wavelets, Mexican hat and Morlet, are capable to achieve a good K-complex detection taken in account the morphology, frequency content, time duration and power spectrum of the K-complex. From this test, the most important conclusion we could extract was that the wavelet capability in the detection of K-complex has a strong dependence on the wavelet waveform. Since the waveform of the wavelet has probed to be an importance parameter for transient signal detection we would like to left this field open for further analysis based on other different wavelet depending on the application they will be used. The way to use the CWT was a precise bandpass filter – we could obtain a very narrow frequency band or only one pseudo-frequency line without big distortion in the signal shape.

We achieved a very good separation of frequencies in a range 0.5 – 3.5 Hz (17 frequency lines) and very good signal suppression in the exterior from this frequency range. This feature of CWT was implemented in both algorithms to detect K-complex signals and was achieved a good results to detect them. To know the real capacity of the algorithms to detect K-complex, they were tested using a single channel from eight hours EEG signal. From the indices specificity, sensitivity and validity we obtained very different results. The performance of the algorithm based on the energy distribution was relatively poor to make a good discrimination between real K-complexes and false K-complexes. The lack of enough criteria for K-complex detection could be the answer of this poor performance. During our experience we realized that the decision regarding the detection of a K-complex may need to be corroboration by a single consideration that we did not take in account. This consideration is concerning to the vicinity of sleep spindles and K-complexes. Another interesting point to mention was the fact that detection of K-complexes was based on the research of only real K-complexes since from the results obtained we realized that a more difficult task to carry out would be the develop of accurate criteria in order to achieve a better recognition between Delta activity and K-complex. When looking in the false K-complexes detected as K-complexes we realized that is possible to find real K-complexes in this set of signals. Almost all these signals are out of stage two, and some of them just in the edge of a particular stage two. This makes to use think that we found real K-complexes in these signals, and a deeper investigation should be made on this field. One possible reason for this problem is that we only looked for K-complexes in stage 2, since we did not find one single reference about the existence of K-complexes out of stage 2. Another reason is a possible not proper stage classification. Even when all signals in question were real K-complexes, the performance of the algorithm will not be good enough, therefore, a criterion for make the difference between K-complexes and Delta waves is highly necessary in order to improve the validity of the algorithms.

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